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ANGLES, MEASURES.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE (1) ANGLE MEASUREMENT, (2) ANGLES AND TRIANGLES, (3) KINDS OF ANGLES, (4) MEASURING THE INTERIOR AND EXTERIOR ANGLES OF POLYGONS, (5) INSCRIBED ANGLES, AND (6) LINES AND ANGLES. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

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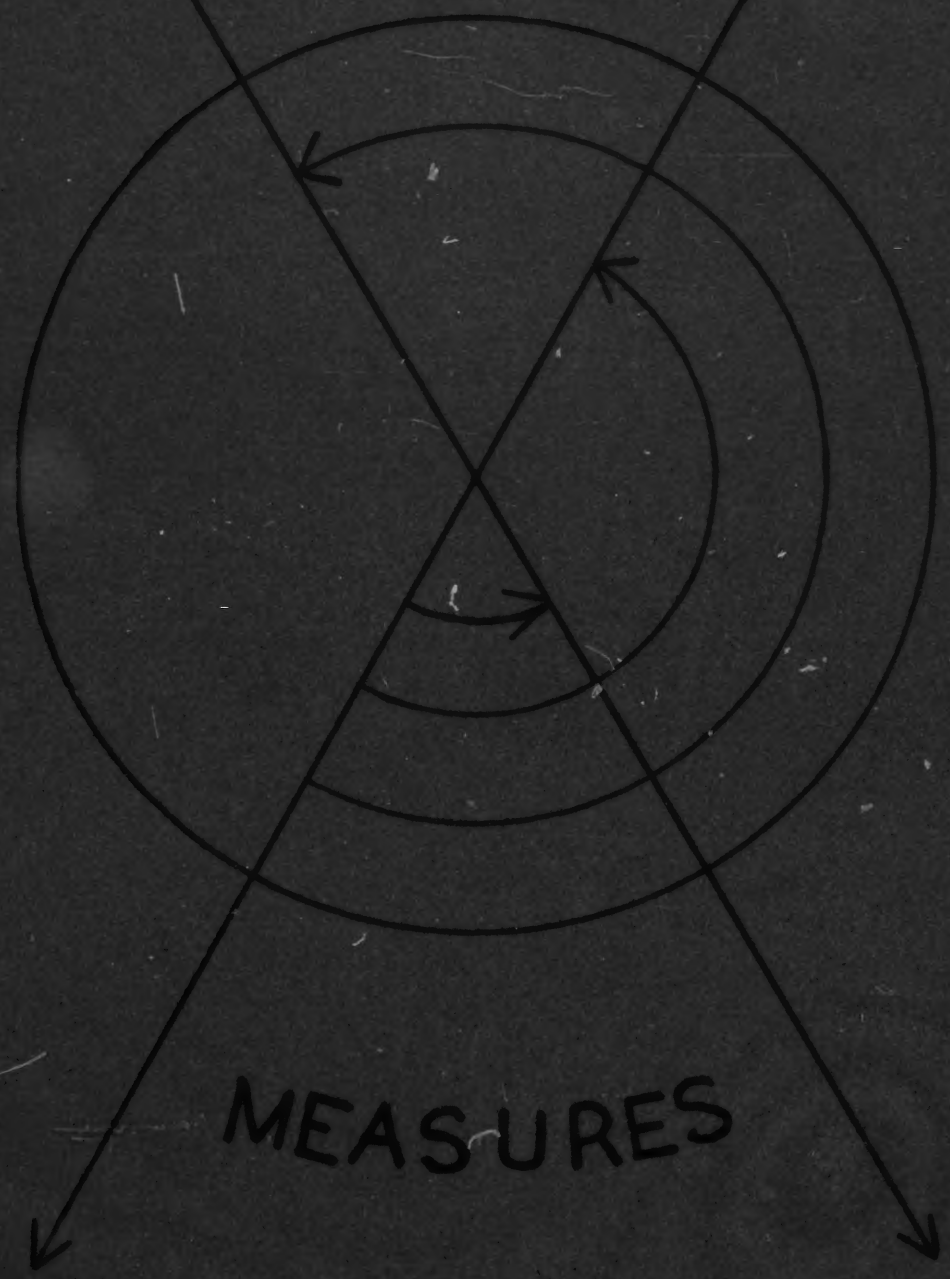
ANGLES



MEASURES

SE 004 518

ESEA Title III

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ANGLES

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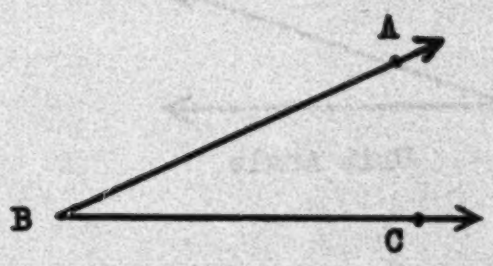
ANGLES

Definitions

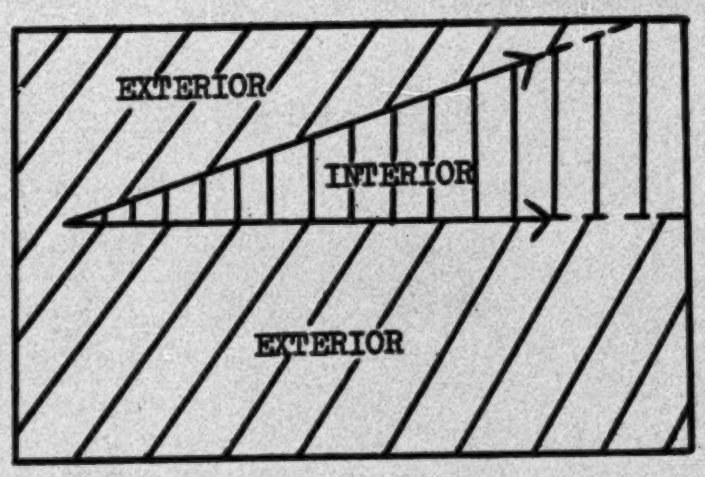
Below is the picture of an angle. Technically, it is the union of two rays with a common end point.



(Fig. 1)

The two rays are symbolized as $\overrightarrow{BA}$ and $\overrightarrow{BC}$ and the angle as $\angle ABC$, $\angle CBA$, or $\angle B$. The rays are called the sides of the angle, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in this angle. The point of intersection of the rays, point B for this angle, is called the vertex.

An angle separates the plane into three sets of points, the interior of the angle, the exterior of the angle, and the angle.



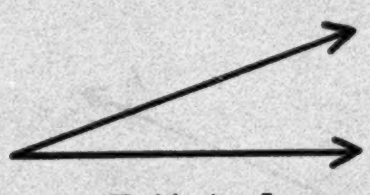
(Fig. 2)

Measuring Angles

When a linear measurement is made, a unit of length is used. What do you think would be used as a unit of measure for an angle? If you said another angle, you were correct.

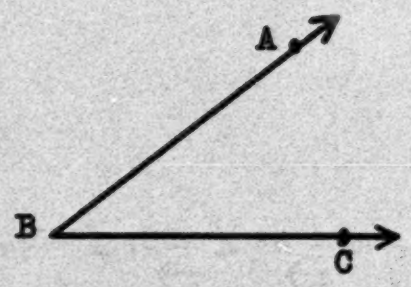
Activities

In each of the problems 1-5, use the given angle as the unit of measure and measure each angle. Remember that measurement is never exact, although it may involve only a very small error, and give your answer to the nearest unit of measure. (Copy the unit angle on another sheet of paper and use your ruler to tear it out.)



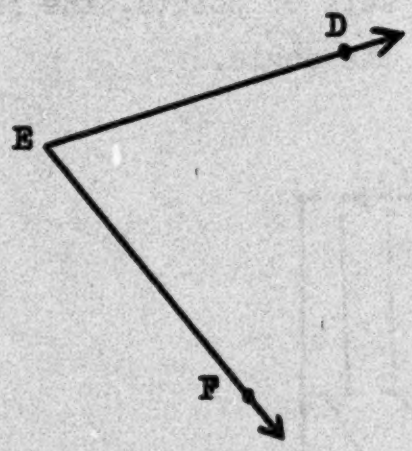
Unit Angle

1.



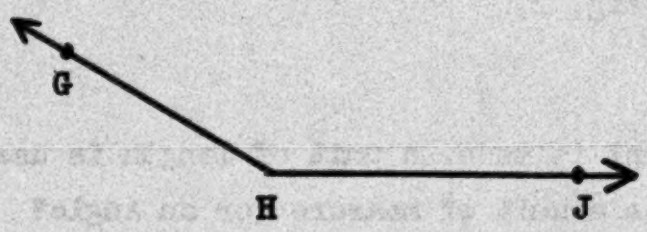
m ∠ABC = _____ (m ∠ABC means the measure of ∠ABC)

2.



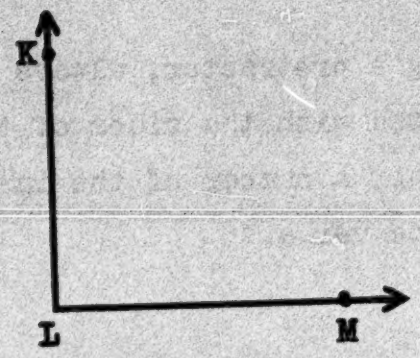
m ∠DEF = _____

3.



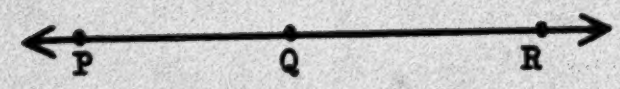
m ∠GHJ = _____

4.



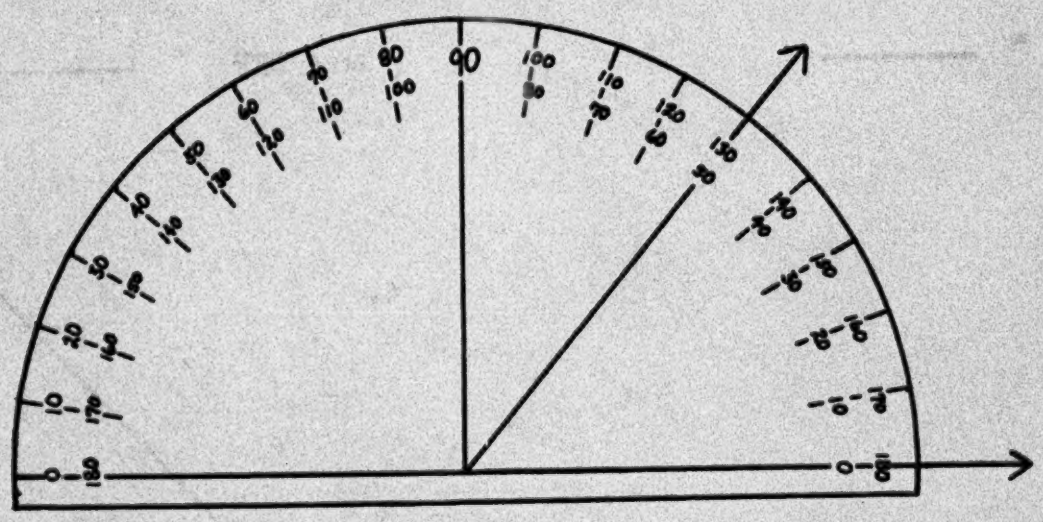
m ∠ KLM = _____.

5.



m ∠ PQR = _____.

Just as the unit measure for length has been standardized, the unit of measure for angles has also been standardized. The two most often used units of measure for angles are the degree and the radian. For our purposes we will use the degree as the unit of measure primarily because an instrument for measuring angles in degrees is readily available. This instrument is called a protractor. The following illustration will show you how to measure an angle, in degrees, with a protractor.



(Fig. 3)

The measure of the angle pictured in figure 3 is 50. That is, it contains 50°. The measure of the angle is found in this manner:

$$50 - 0 = 50$$

or

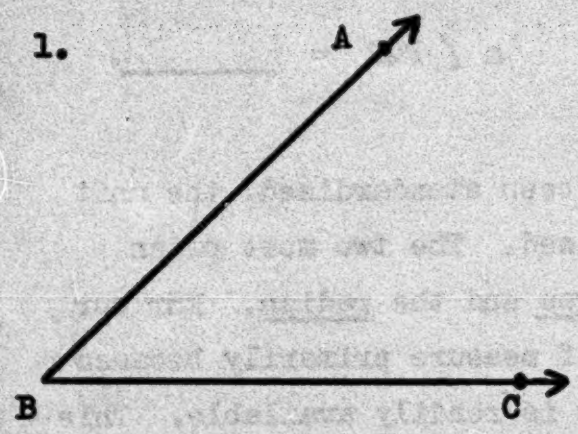
$$180 - 130 = 50$$

When finding the measure of an angle using a protractor, always use the two outside numerals which are associated with the sides of the angle or the two inside numerals. Never subtract a number of the inside scale from a number of the outside scale or vice versa.

Activities

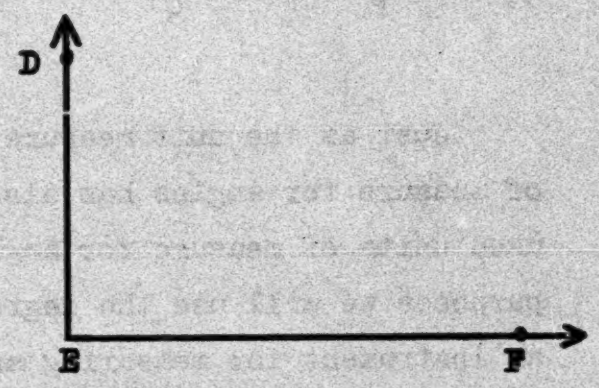
With the use of a protractor, find the measures of the following angles. (Give your answer to the nearest degree. We will not be measuring any angle whose measures are greater than 180 .)

1.



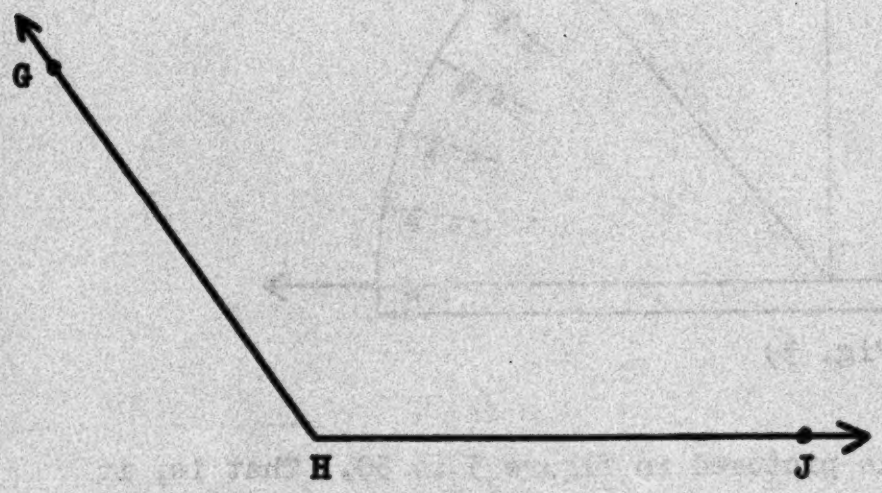
m ∠ABC = _____

2.



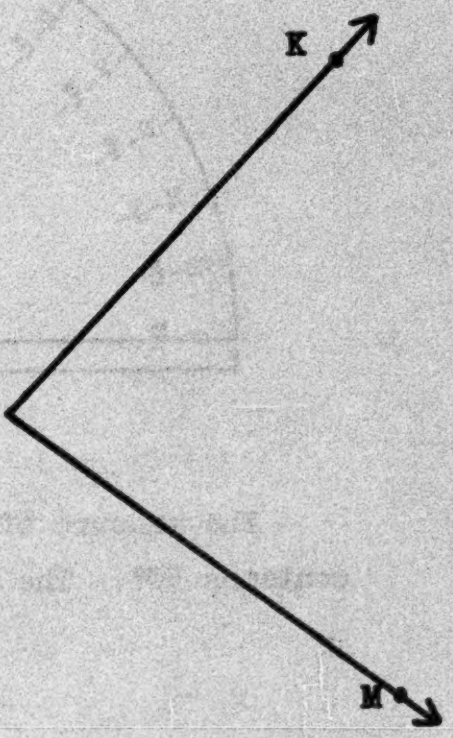
m / DEF = _____

3.



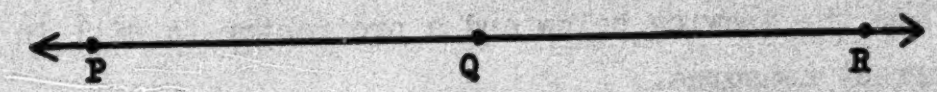
m ∠GHJ = _____

4.



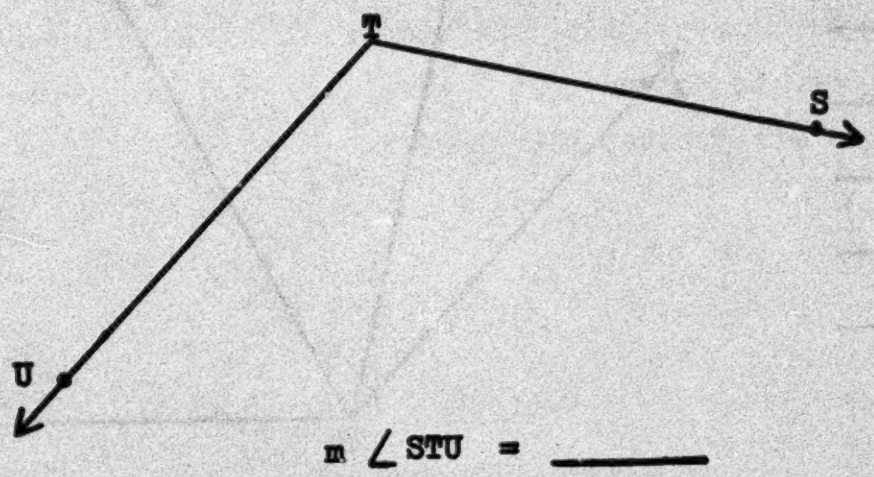
m ∠KLM = _____

5.



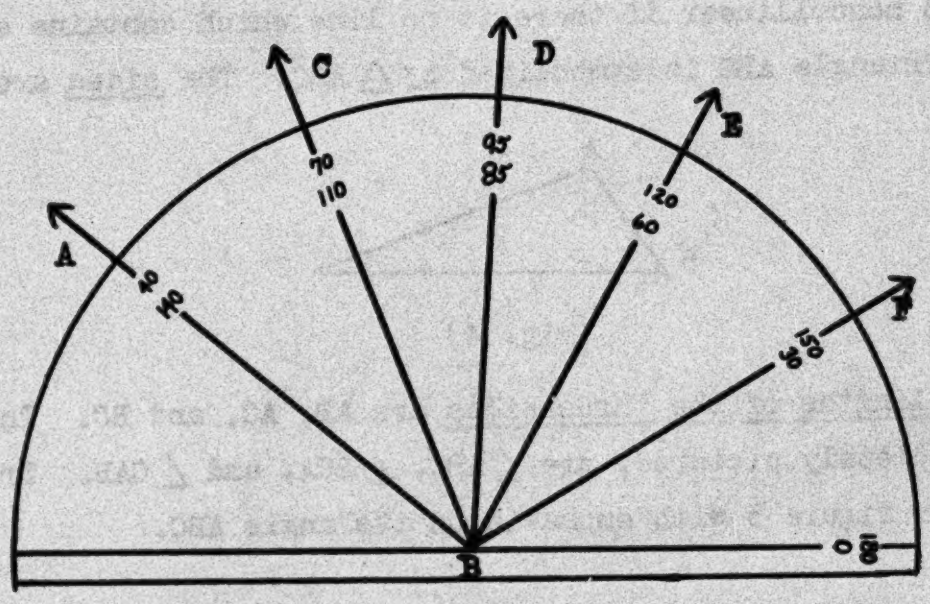
m ∠ PQR = _____

6.



m ∠ STU = _____

Use the drawing below to find the measures of the indicated angles. Problem number 7 is an example.



7. m ∠ ABC = 140 - 110 = 30

8. m ∠ CBD = 110 - 85 = _____

9. m ∠ DBE = 85 - 60 = _____

10. m ∠ EBF = _____ = _____

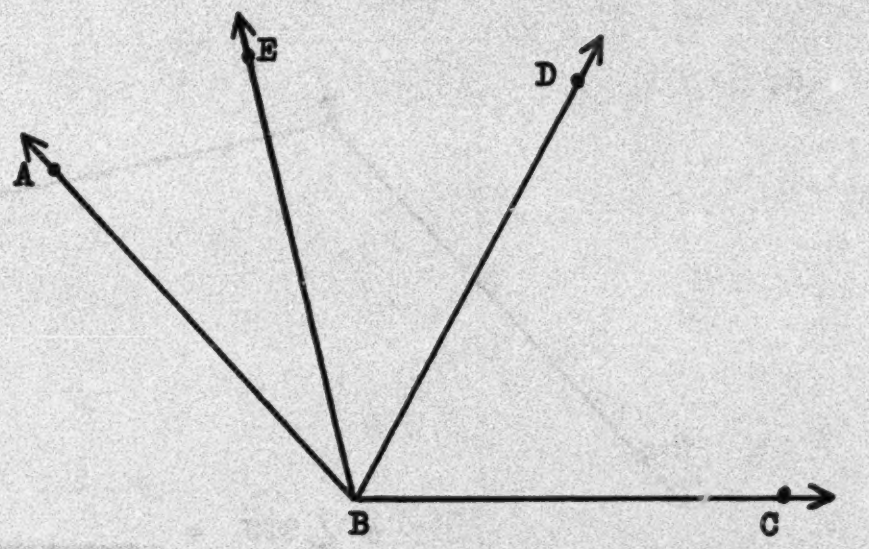
11. m ∠ ABD = _____ = _____

12. m ∠ ABE = _____ = _____

13. m ∠ ABF = _____ = _____

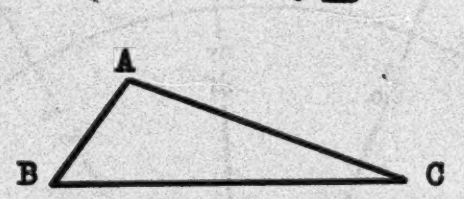
In problems 14-18 use the drawing below and a protractor to fill in the blanks with the correct measures.

14. m ∠ ABC = _____
15. m ∠ ABE = _____
16. m ∠ EBD = _____
17. m ∠ ABD = _____
18. m ∠ DBC = _____

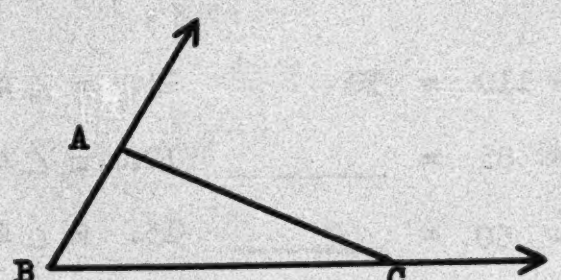


Angles and Triangles

The figure below is a picture of a triangle. Technically, a triangle is the union of three line segments with three noncollinear points. The three noncollinear points are represented by A, B, and C in this drawing. (Three points are noncollinear if there is no line which contains all three points.) Triangle ABC is symbolized by △ ABC. The <u>sides</u> are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. <u>The lengths of the three sides</u> are AB, AC, and BC. The angles, which are not completely pictured, are ∠ ABC, ∠ BCA, and ∠ CAB. Triangle ABC is pictured in figure 5 with emphasis on its angle ABC.



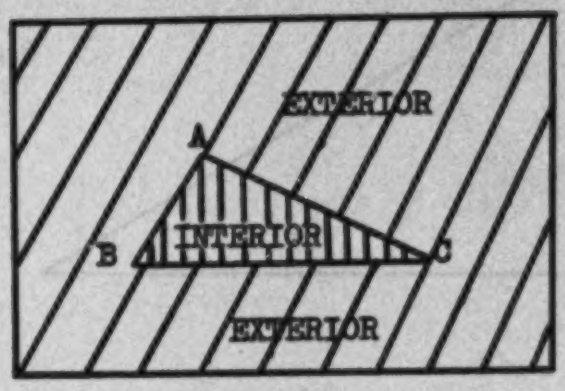
(Fig. 4)



(Fig. 5)

A triangle separates the plane into three sets of points, the interior of the triangle, the exterior of the triangle, and the triangle. The

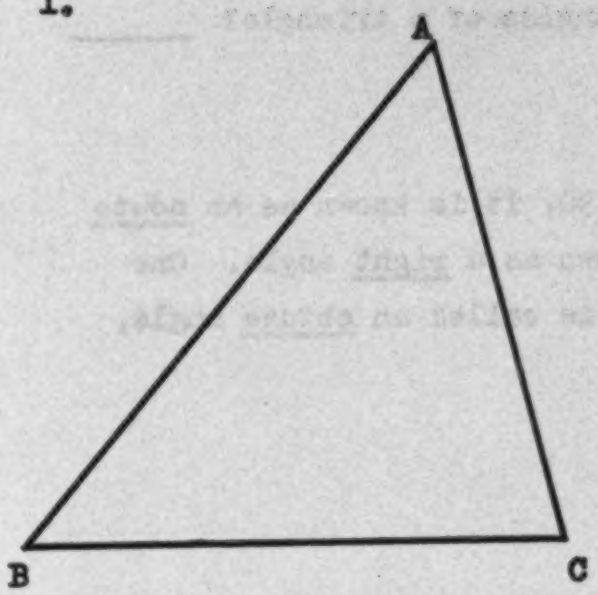
interior of the triangle and the triangle form what is known as a triangular region. That is, the union of the triangle and its interior is a triangular region.



(Fig. 6)

In problems 1-3 find the measures of each of the angles of the triangles. (Use your protractor.)

1.

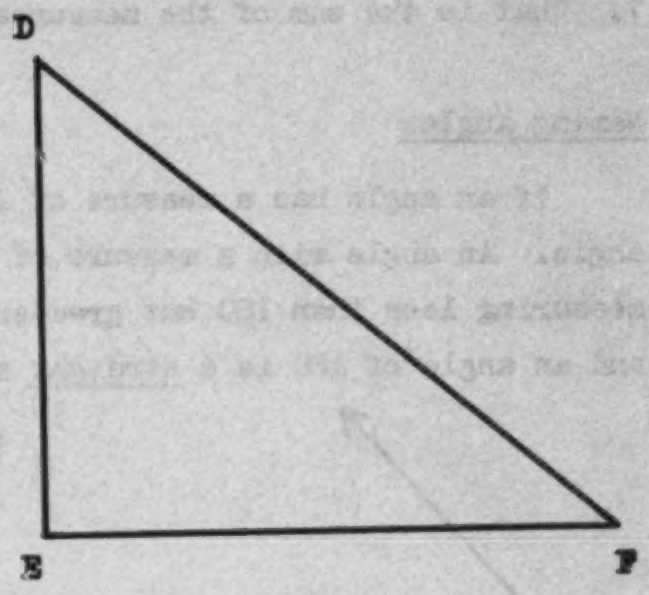


$m\angle A$ = ____

$m\angle B$ = ____

$m\angle C$ = ____

2.

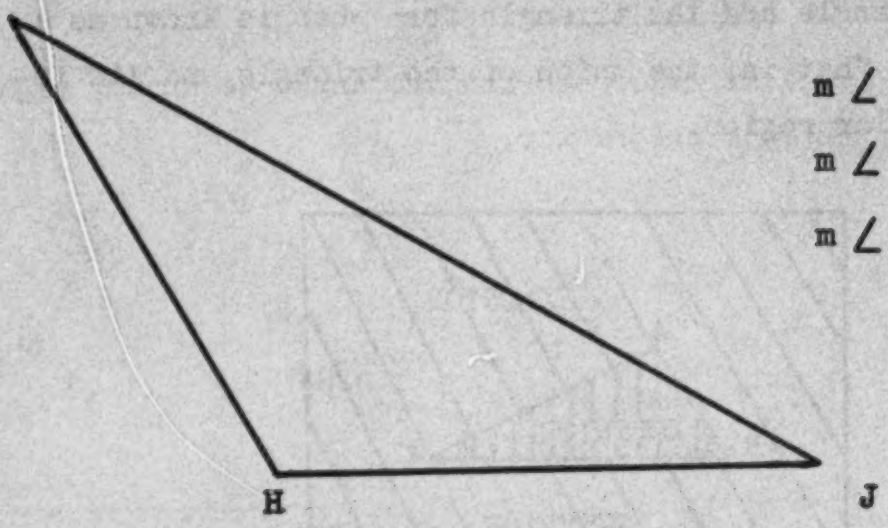


$m\angle D$ = ____

$m\angle E$ = ____

$m\angle F$ = ____

3.



m ∠G = _____

m ∠H = _____

m ∠J = _____

Use problems 1- 3 to answer each of these:

4. m ∠A + m ∠B + m ∠C = _____.

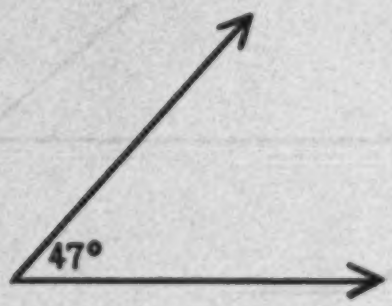
5. m ∠D + m ∠E + m ∠F = _____.

6. m ∠G + m ∠H + m ∠J = _____.

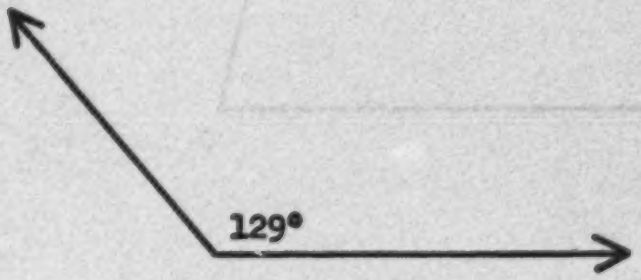
7. What is the sum of the measures of the angles of a triangle? _____

Naming Angles

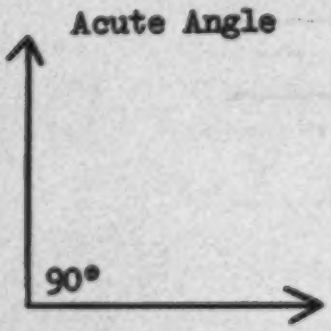
If an angle has a measure of less than 90, it is known as an acute angle. An angle with a measure of 90 is known as a right angle. One measuring less than 180 but greater than 90 is called an obtuse angle, and an angle of 180 is a straight angle.



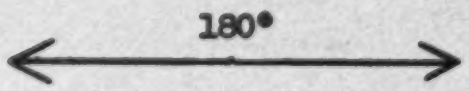
Acute Angle



Obtuse Angle



Right Angle

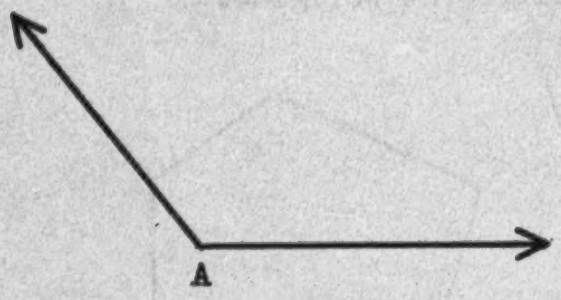


Straight Angle

Activities

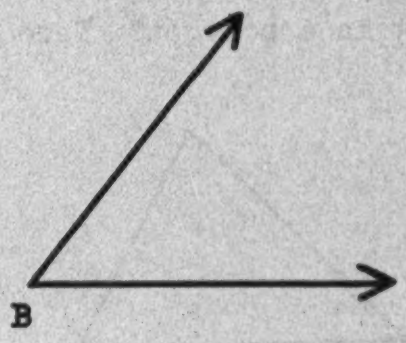
In each of the problems 1-10 describe the angle as acute, right, or obtuse. You may use your protractor.

1.



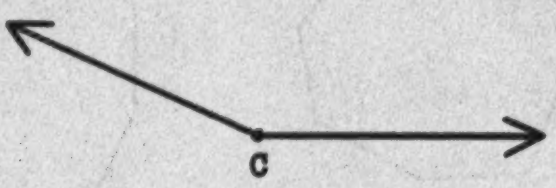
∠A is _____.

2.

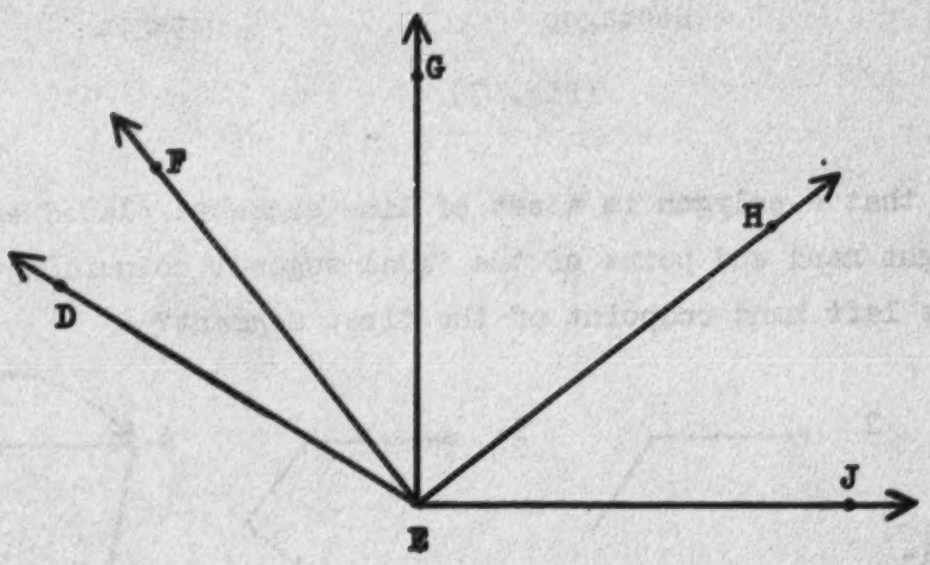


∠B is _____.

3.



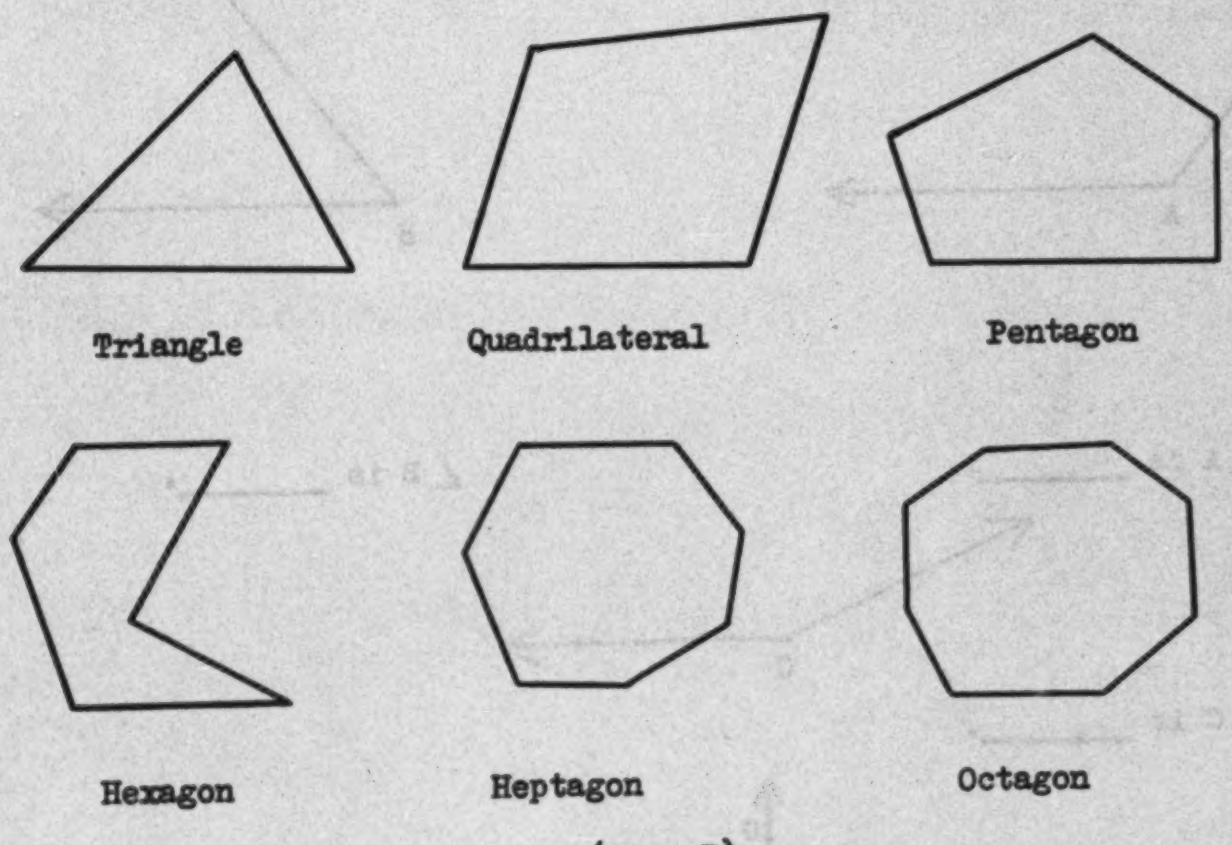
∠C is _____.



4. ∠DEF is _____.
5. ∠DEG is _____.
6. ∠DEH is _____.
7. ∠DEJ is _____.
8. ∠FEG is _____.
9. ∠FEH is _____.
10. ∠GEJ is _____.

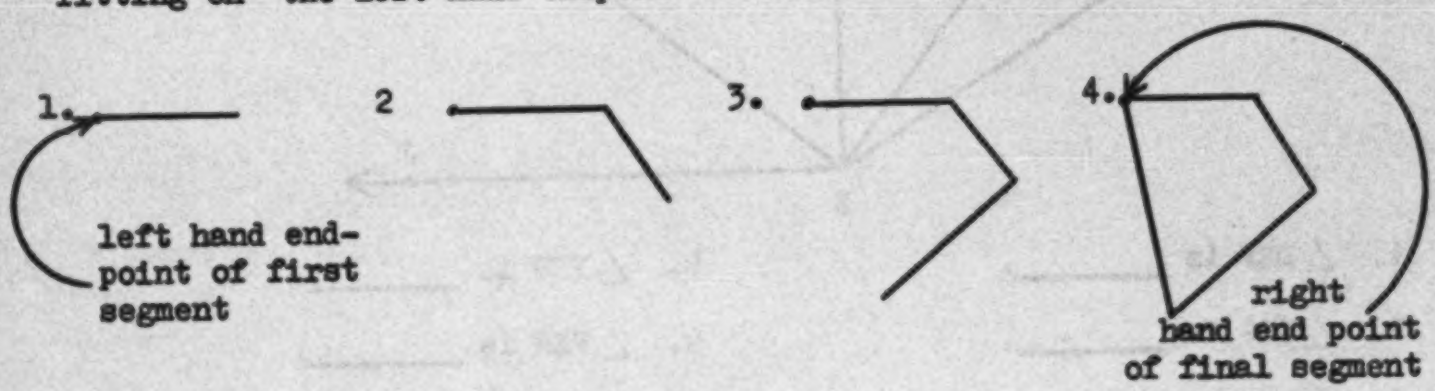
Polygons and the Measures of Their Angles

We have just seen that the sum of the measures of the angles of a triangle is 180. Let us use this fact to determine the sum of the measures of the angles of a polygon of more than three sides. First, what is a polygon? Here are pictures of six different polygons.



(Fig. 7)

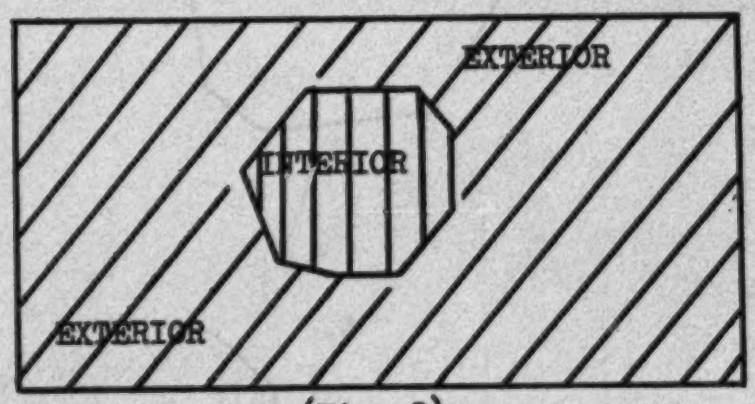
Do you see that a polygon is a set of line segments placed end to end with the right hand end point of the final segment coinciding or "fitting on" the left hand endpoint of the first segment?



Building a Polygon (quadrilateral)

The hexagon of figure 7 is a concave polygon. The other polygons are convex.

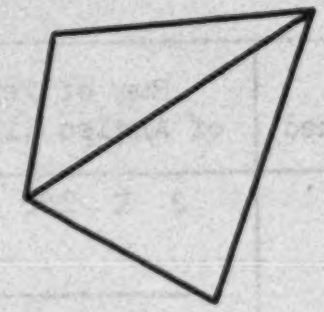
Any convex polygon and some concave polygons separate the plane into three sets of points, the interior of the polygon, the exterior of the polygon, and the polygon.



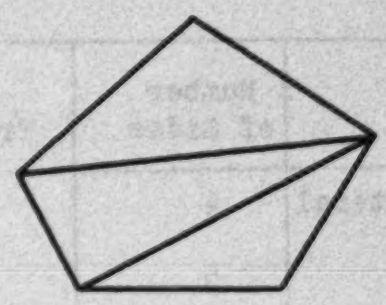
(Fig. 8)

The union of the polygon and its interior is called a polygonal region.

The number of degrees in the angles of any polygon may be determined by separating it into triangles. This is illustrated below.



(Fig. 9)



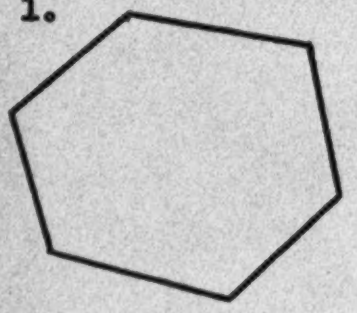
(Fig. 10)

Two triangles are formed in figure 9, and the sum, in degrees, of the measures of the angles is 2 X 180, or 360. What is the sum of the measures of the angles of the polygon pictured in figure 10? _____

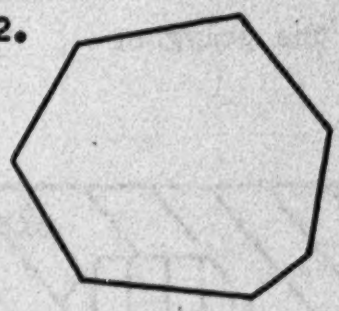
Activities

In each of the problems 1-5 find the sum of the measures of the angles of the indicated polygon.

1.



2.



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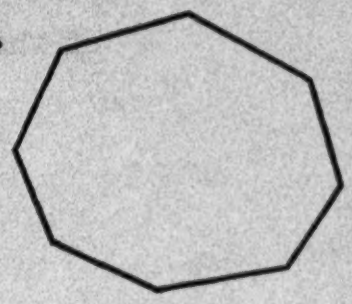
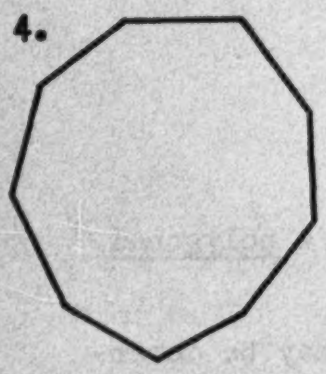
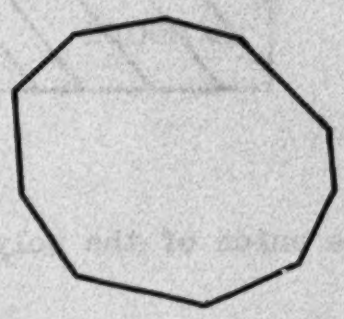


4.



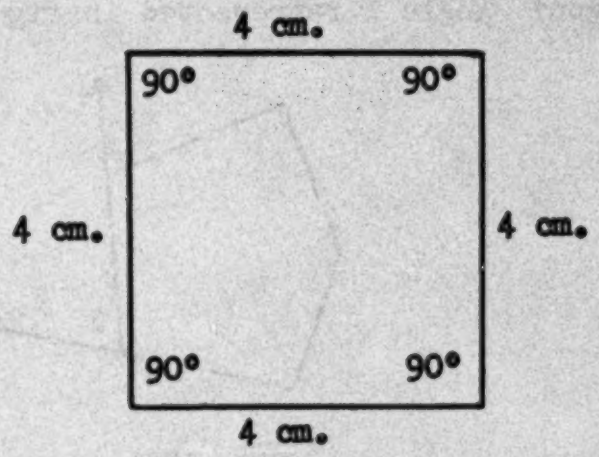
5.



6. Complete the following table and see if you can determine the number of degrees in the angles of any polygon.

Name of Polygon	Number of Sides	Number of Triangles Formed	Sum of Measures of Angles (in Degrees)
Quadrilateral	4	2	2 X 180 = 360
Pentagon	5	3	_____
Hexagon	6	____	_____
Heptagon	7	____	_____
Octagon	8	____	_____
Nonagon	9	____	_____
Decagon	10	____	_____
n-gon	n	____	_____

A <u>regular</u> polygon is a polygon whose sides all have the same length and whose angles have the same measure. For instance, the quadrilateral which is pictured below is a regular quadrilateral.



(Fig. 11)

Measure of One Angle of Regular Polygons

Activities

1. How would you determine the number of degrees in one angle of a regular polygon? Complete the table below and see if you can answer the previous question?

Name of Polygon (Regular)	Sum of Measures of Angles (in Degrees)	No. Angles	No. of Degrees in One Angle
Square	360	4	360 ÷ 4 = 90
Regular Pentagon	540	5	_____
Regular Hexagon	____	____	_____
Regular Heptagon	____	____	_____
Regular Octagon	____	____	_____
Regular Nonagon	____	____	_____
Regular Decagon	____	____	_____

2. Find the number of degrees in one angle of a regular 20-gon.

3. Find the number of degrees in one angle of a regular 25-gon.

By now it is apparent that the sum of the measures of the angles of a polygon of 20 sides is greater than the sum of the measures of the angles of a polygon of 19 sides.

Measure of Exterior Angles of Polygon

What about the sum of the measures of the exterior angles, one at each vertex, of a polygon? Angle 1 represented in figure 12 is an exterior angle.

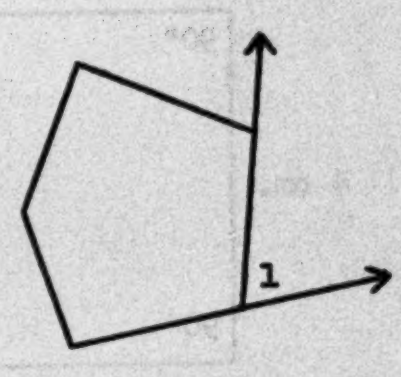
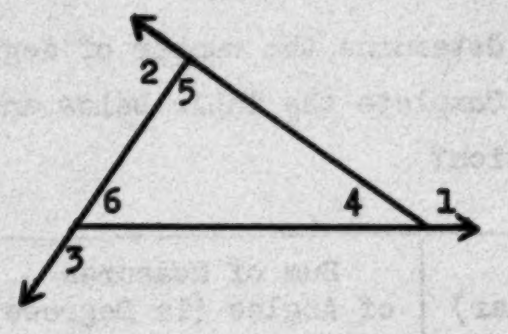


Fig. 12)

Consider first the exterior angles, one at each vertex, of a triangle.



(Fig. 13)

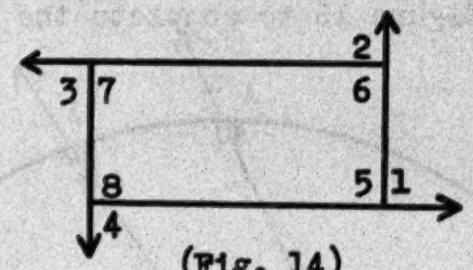
The angle pairs 1 and 4, 2 and 5, and 3 and 6 each form a straight angle.

Activities

Find the sum of each of the following: (m ∠ means measure of the angle in degrees.)

1. m ∠1 + m ∠4 = _____.
2. m ∠2 + m ∠5 = _____.
3. m ∠3 + m ∠6 = _____.
4. m ∠1 + m ∠2 + m ∠3 + m ∠4 + m ∠5 + m ∠6 = _____.
5. m ∠4 + m ∠5 + m ∠6 = _____; m ∠1 + m ∠2 + m ∠3 = _____.

Use figure 14 to solve problems 6-13.



(Fig. 14)

6. m ∠1 + m ∠5 = _____.

7. m ∠2 + m ∠6 = _____.

8. m ∠3 + m ∠7 = _____.

9. m ∠4 + m ∠8 = _____.

10. m ∠1 + m ∠2 + m ∠3 + m ∠4 + m ∠5 + m ∠6 + m ∠7 + m ∠8 = _____.

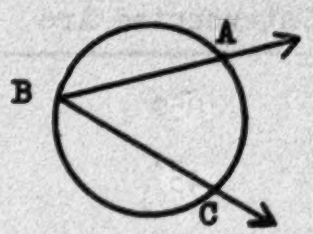
11. m ∠5 + m ∠6 + m ∠7 + m ∠8 = _____.

12. m ∠1 + m ∠2 + m ∠3 + m ∠4 = _____.

13. What is the sum of the measures of the exterior angles of a quadrilateral? _____ An octagon? _____ A 20-gon? _____.

Inscribed Angles

There is a relationship between what is known as an inscribed angle and its intercepted arc. Figure 15 illustrates an inscribed angle.

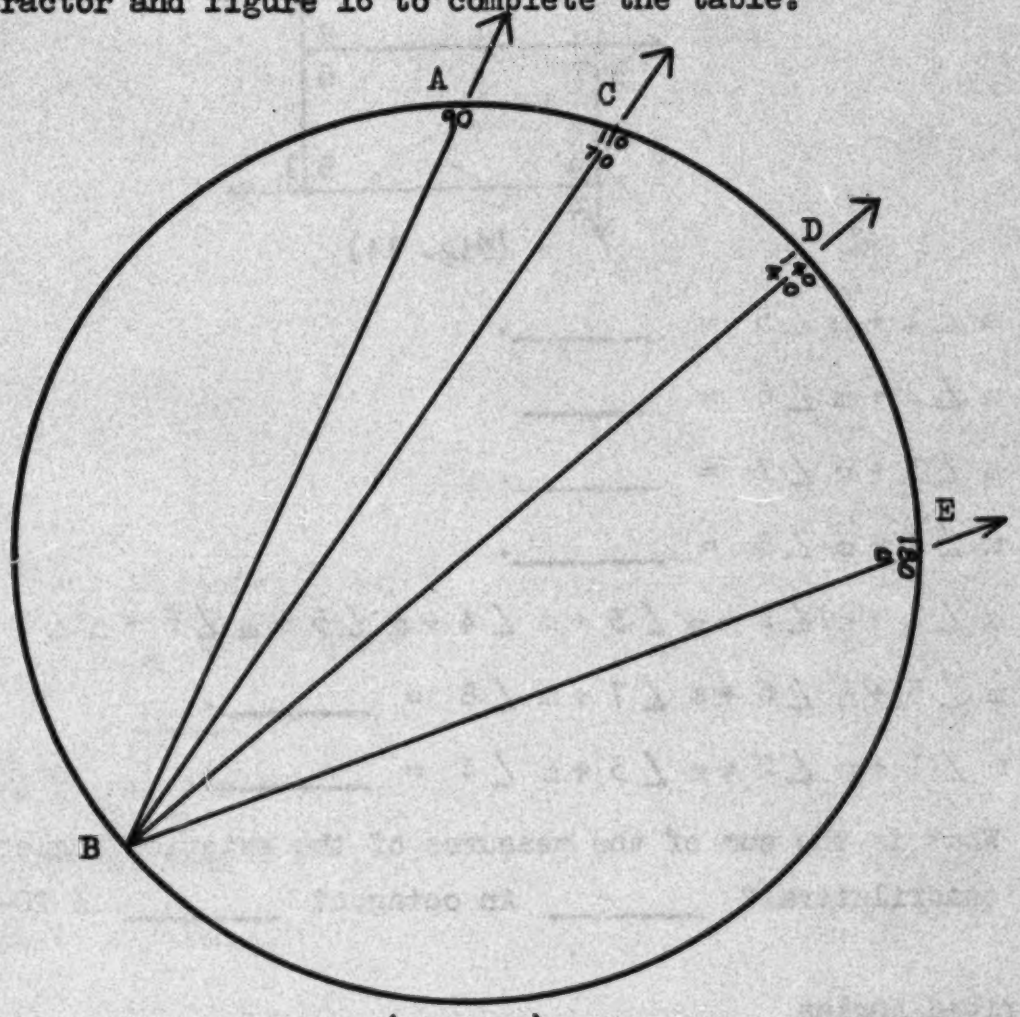


(Fig. 15)

To be an inscribed angle, an angle must meet the following requirements: the sides of the angle must contain two chords of the circle, in this case $\overline{AB}$ and $\overline{BC}$; the vertex of the angle, B for this angle, must be a point of the circle. The part of the circle not in the exterior of the angle, $\overset{\frown}{AC}$ here, is the intercepted arc.

Activities

1. Use your protractor and figure 16 to complete the table.

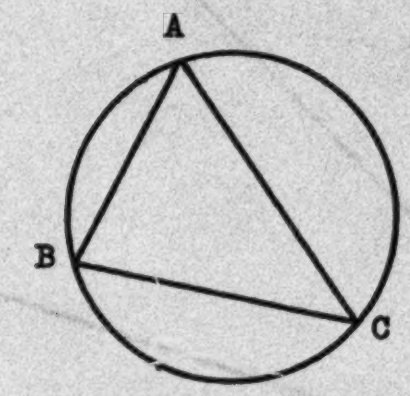


(Fig. 16)

Angle	Intercepted Arc	Number of Degrees in Intercepted Arc	Number of Degrees In Inscribed Angle
∠ ABE	$\overset{\frown}{AE}$	90	_____
∠ ABD	$\overset{\frown}{AD}$	50	_____
∠ ABC	$\overset{\frown}{AC}$	20	_____
∠ CBE	$\overset{\frown}{CE}$	70	_____
∠ CBD	$\overset{\frown}{CD}$	30	_____
∠ DBE	$\overset{\frown}{DE}$	40	_____

2. What is the relationship between an inscribed angle and its intercepted arc? _____

Use the relationship between an inscribed angle and its intercepted arc to answer questions 3-9. The questions refer to figure 17.



(Fig. 17)

3. What is the sum, in degrees, of the measures of the three arcs, $\overset{\frown}{AC}$, $\overset{\frown}{CB}$, and $\overset{\frown}{BA}$? _____

4. Find $\frac{1}{2}$ of the measure of $\overset{\frown}{AC} + \overset{\frown}{CB} + \overset{\frown}{BA}$. _____

5. The measure of $\angle ABC$ equals one-half the measure of arc _____.

6. The measure of $\angle BAC$ equals one-half the measure of arc _____.

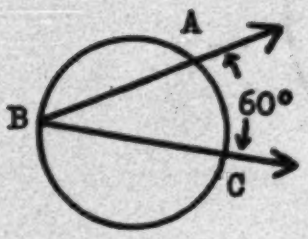
7. The measure of $\angle ACB$ equals one-half the measure of arc _____.

8. $m\angle ABC + m\angle BAC + m\angle ACB = \frac{1}{2}\, m\,(\overset{\frown}{AC} + ___ + ___)$.

9. $m\angle ABC + m\angle BAC + m\angle ACB =$ _____ degrees.

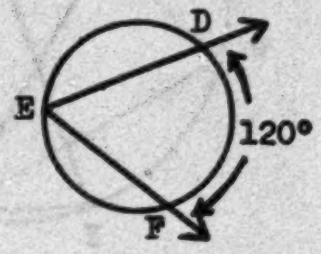
In each of the problems 10-14 determine the measure of the angle or angles without using your protractor:

10.



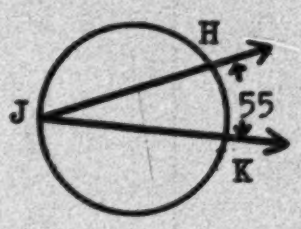
$m\angle ABC =$ _____.

11.



$m\angle DEF =$ _____.

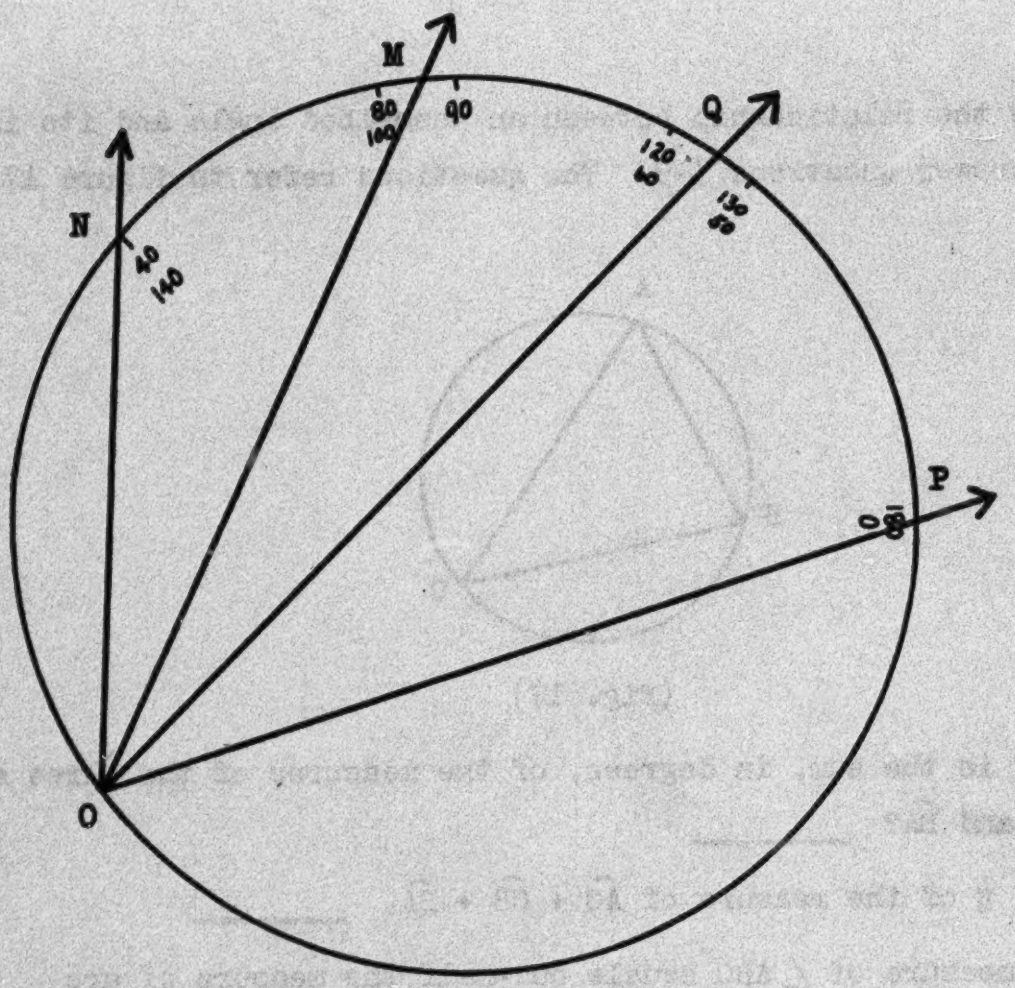
12.



$m\angle HJK =$ _____.

18

13.



m ∠MOP = _____°

m ∠NOP = _____°

m ∠QOP = _____°

m ∠MOQ = _____°

m ∠NOM = _____°

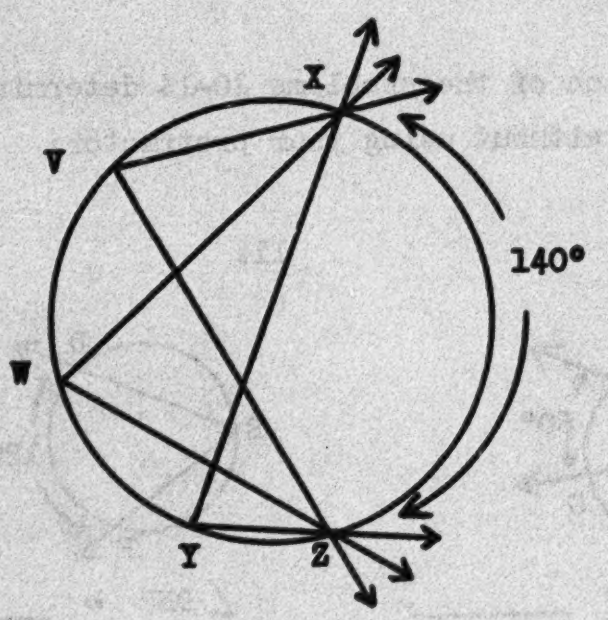
m ∠NOQ = _____°

14.

m ∠XYZ = _____°

m ∠XWZ = _____°

m ∠XVZ = _____°



15. The three angles XVZ, XWZ, and XYZ are three inscribed angles which intercept the same arc. Are inscribed angles which intercept the same arc always equal? _____

In each of the triangles represented by figures 18-20 $\angle ABD$ is an <u>exterior angle</u> of the triangle. The angles A and C are the <u>remote interior angles</u> with respect to the angle ABD.

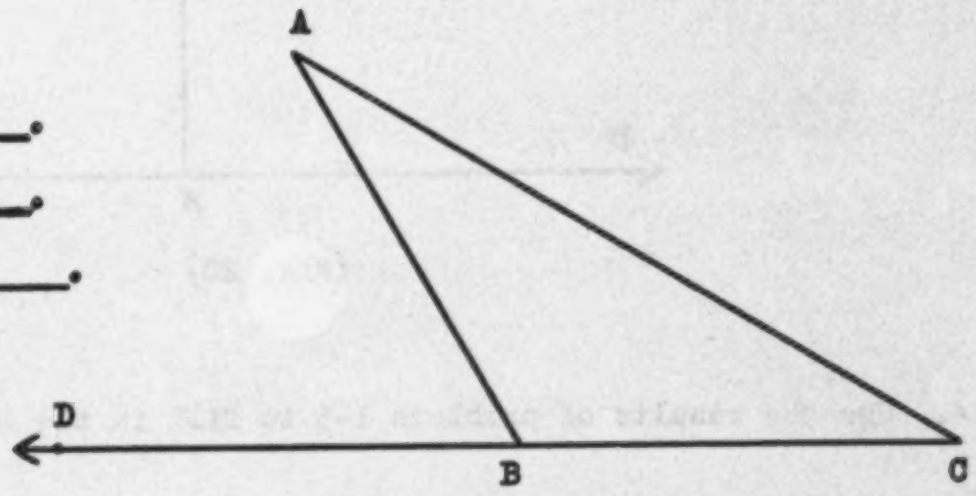
Activities

With your protractor find the measures of $\angle ABD$, $\angle A$, and $\angle C$ in each of the triangles represented in figures 18-20.

1. $m\angle A$ = _____.

   $m\angle C$ = _____.

   $m\angle ABD$ = _____.

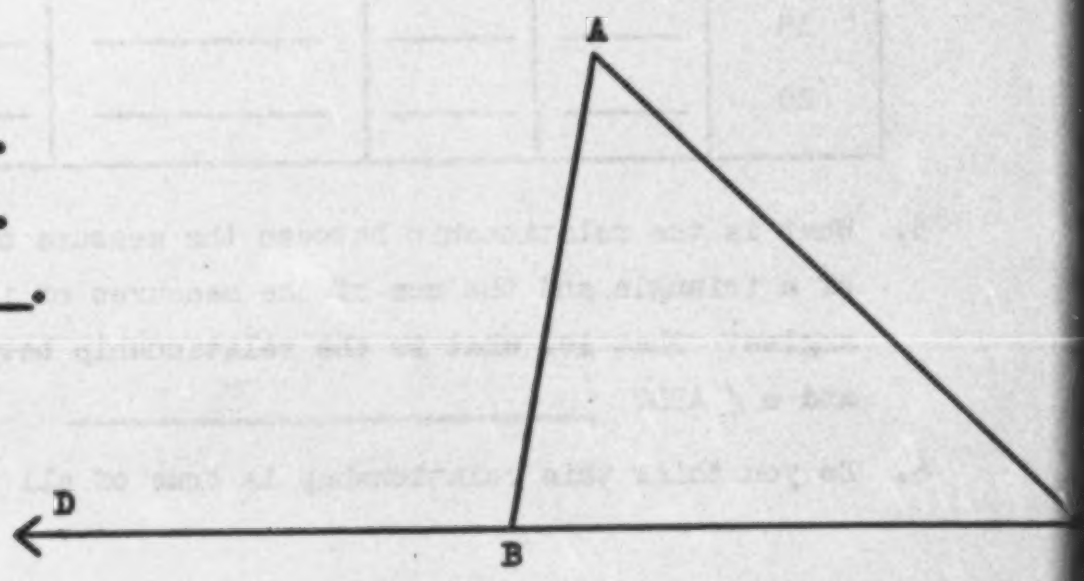


(Fig. 18)

2. $m\angle A$ = _____.

   $m\angle C$ = _____.

   $m\angle ABD$ = _____.

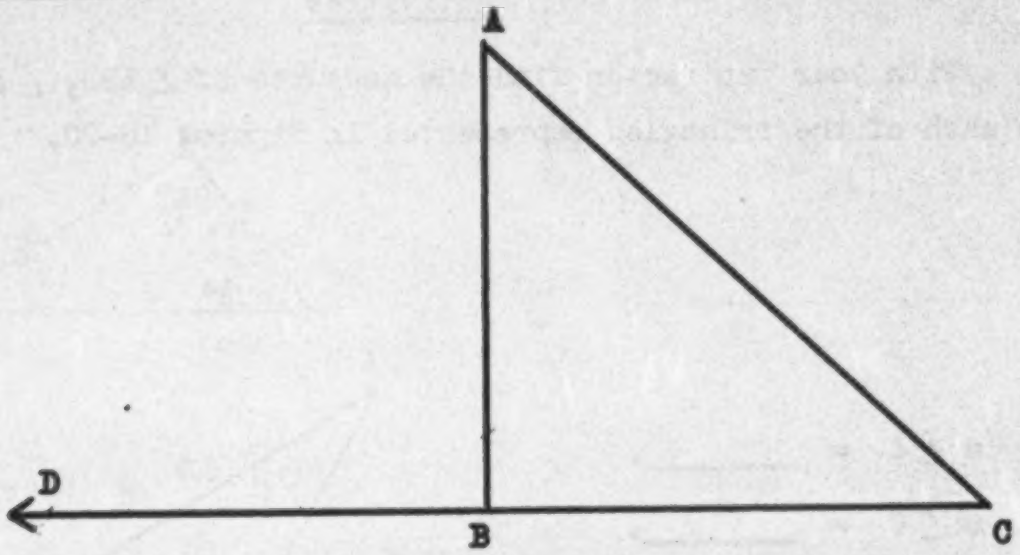


(Fig. 19)

3. m ∠A = _____.

   m ∠C = _____.

   m ∠ABD = _____.



(Fig. 20)

4. Use the results of problems 1-3 to fill in the table here.

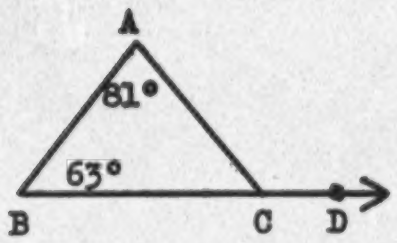
Figure	m ∠A	m ∠C	m ∠A + M ∠C	m ∠ABD
18	_____	_____	_____	_____
19	_____	_____	_____	_____
20	_____	_____	_____	_____

5. What is the relationship between the measure of the exterior angle of a triangle and the sum of the measures of its two remote interior angles? That is, what is the relationship between m ∠A + m ∠C and m ∠ABD? _____

6. Do you think this relationship is true of all triangles? _____

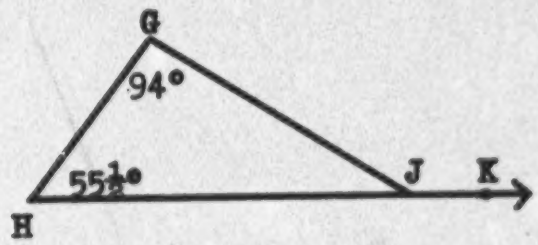
In each of the problems 7-11, find the measure of the exterior angle of each of the triangles by using the fact that the measure of the exterior angle of a triangle equals the sum of the measures of the two remote interior angles.

7.



m ∠ ACD = _____.

8.



m ∠ GJK = _____.

9.

m ∠ QRS = _____.

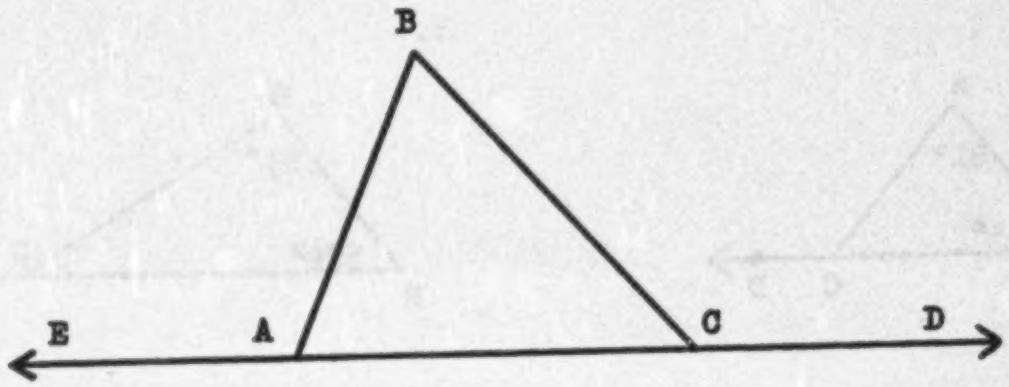
10.

m ∠ EFG = _____.

11.

m ∠ CBE = _____.

You are familiar with the symbols " > " and " < ." We would write 5<6 and read this as " 5 is less than 6." To say that "10 is greater than 9" is to write 10>9. Use one of the three symbols, =, <, or >, to make the following statements true. Angles BCD and BAE are exterior angles.



12. m ∠BCD _____ m ∠BAC

13. m ∠BCD _____ m ∠B + m ∠BAC

14. m ∠B _____ m ∠BCD

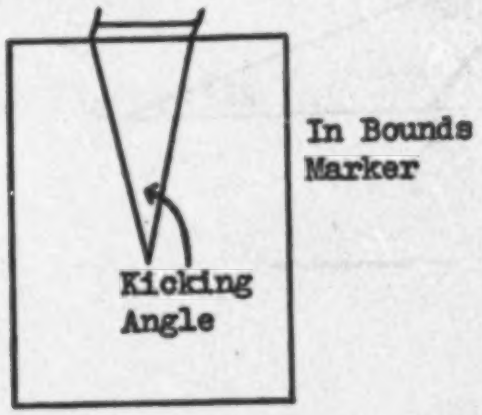
15. m ∠B _____ m ∠BAE

16. m ∠B + m ∠ACB _____ m ∠EAB

17. m ∠BCA _____ m ∠EAB

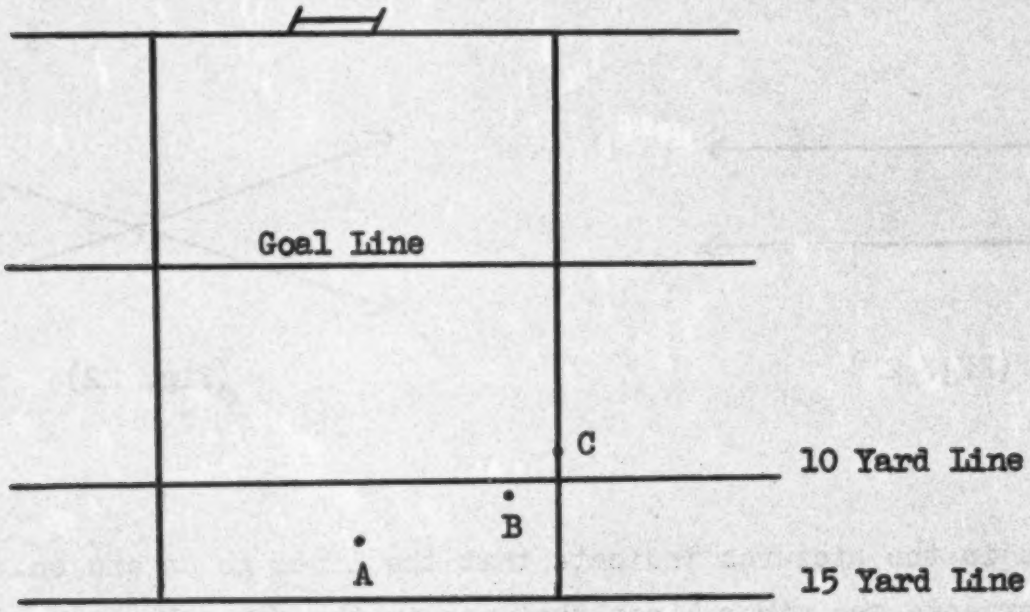
18. Do you think that in any triangle the exterior angle is greater than either of the remote interior angles? _____

Where is the best place on the football field from which to kick a field goal? You would probably agree that there are two primary considerations in kicking this field goal, the distance from the place of kicking and the angle in which the football must travel to be a good kick.

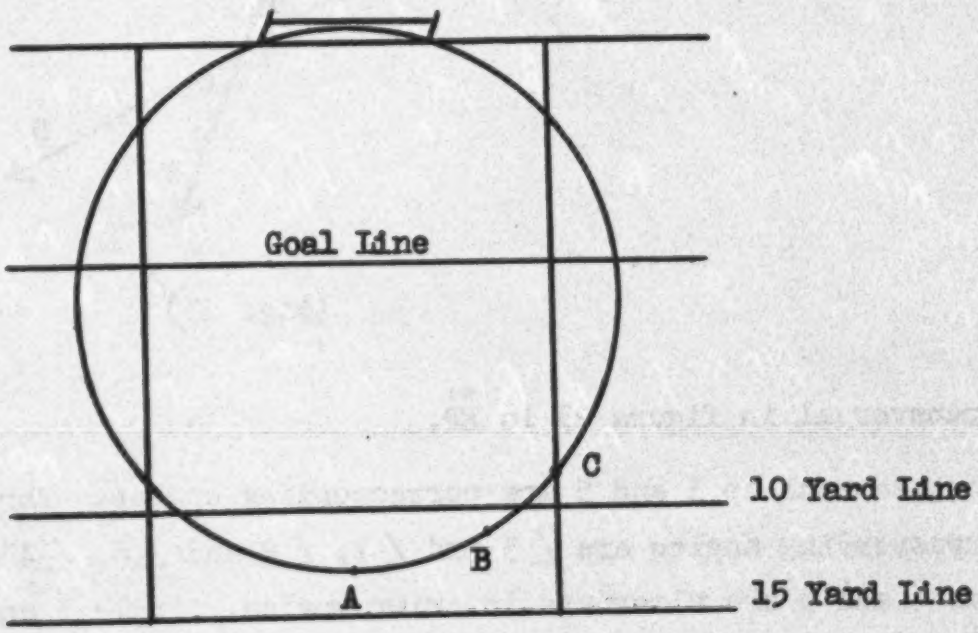


Activities

1. Which one of these positions would be most favorable for kicking a field goal? Give a reason for your answer.



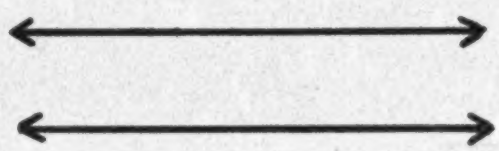
2. This is problem number 1 again with a hint added. Now, do you think your answer to problem 1 is correct? _____ If not, what should be your answer to problem 1? _____



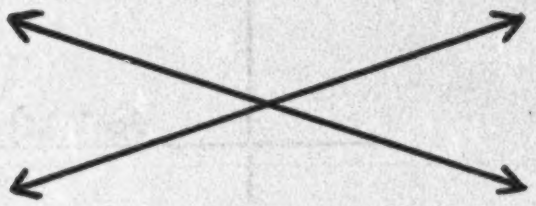
3. If you were kicking a field goal from the 10 yard line, where on the field would be the most favorable kicking angle? _____ Explain.

Lines and Angles

Two lines are parallel if they lie in the same plane and do not intersect or meet. Figure 21 is a picture of two parallel lines and figure 22 is a picture of two non-parallel lines.

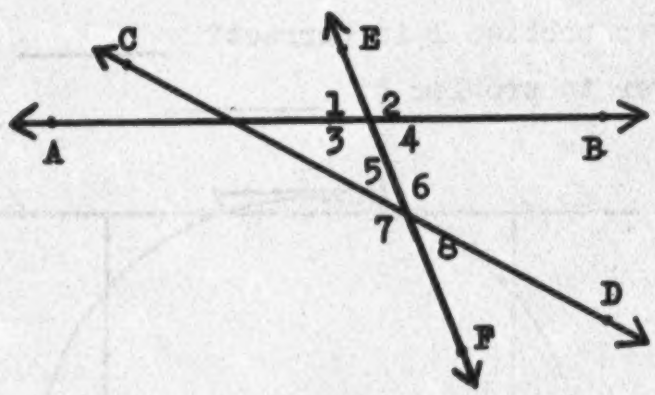


(Fig. 21)



(Fig. 22)

The arrows in the pictures indicate that the lines go on and on. A line which intersects two other lines that are in the same plane is called a transversal.



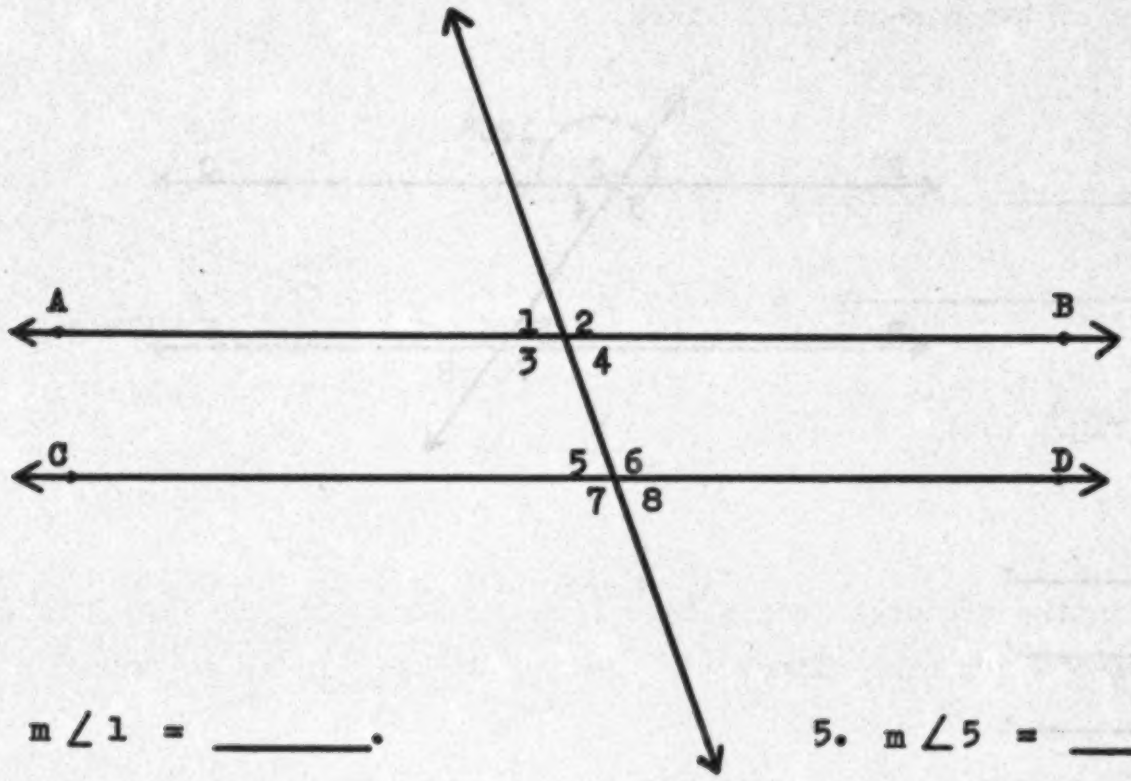
(Fig. 23)

One transversal in figure 23 is $\overleftrightarrow{EF}$.

Angles such as 1 and 5 are corresponding angles. Three other pairs of corresponding angles are $\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$. Angles such as 4 and 5 are alternate interior angles. Angle 3 and angle 6 are also alternate interior angles.

Activities

In each of the problems 1-8 use your protractor to find the measures of angles 1-8. ($\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel.)



1. m $\angle 1$ = _____.
2. m $\angle 2$ = _____.
3. m $\angle 3$ = _____.
4. m $\angle 4$ = _____.
5. m $\angle 5$ = _____.
6. m $\angle 6$ = _____.
7. m $\angle 7$ = _____.
8. m $\angle 8$ = _____.

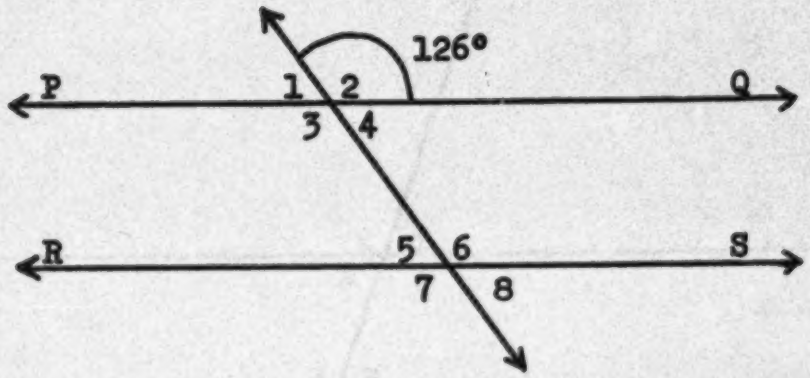
9. Angle 2 and angle 6 are corresponding angles. What do you notice about the measures of these two angles? _____

10. Name three other pairs of corresponding angles. _____ _____ What seems to be true of the measures of each of these pairs of corresponding angles? _____ _____

11. Angle 3 and angle 6 are alternate interior angles. What do you notice about the measures of these two angles? _____ _____

12. Name another pair of alternate interior angles. _____ What seems to be true of the measures of this pair of alternate interior angles? _____

Without using your protractor give the measures of angles 1, 3, 4, 5, 6, 7, and 8. In the drawing, $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are parallel.



m∠1 = _____°.

m∠5 = _____°.

m∠4 = _____°.

m∠6 = _____°.

m∠3 = _____°.

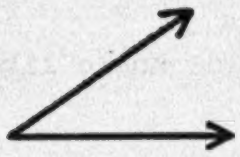
m∠7 = _____°.

m∠8 = _____°.

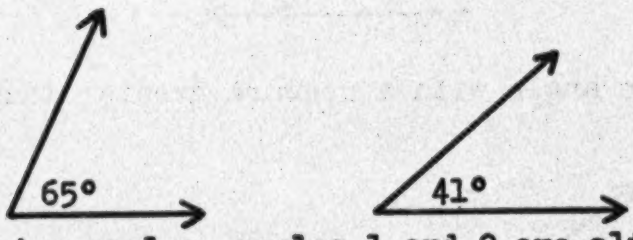
ANGLES

Illustration of Terms

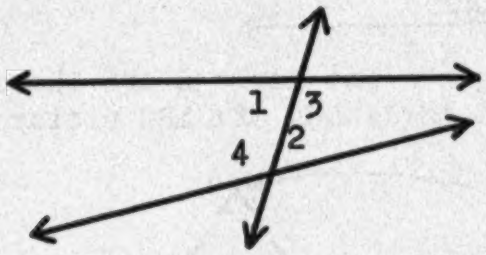
Angle, the plane figure formed by the union of two rays.



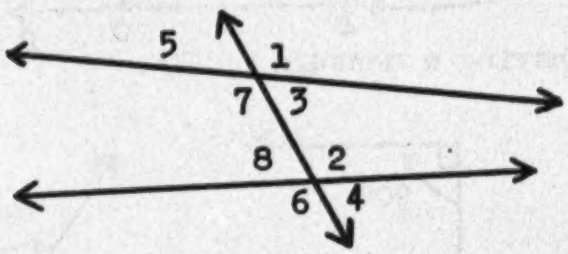
Acute angle, an angle with a measure less than 90 but more than 0.



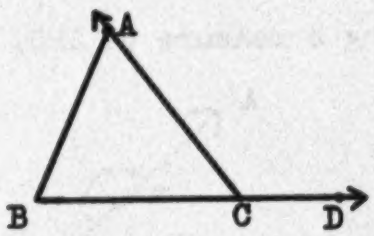
Alternate interior angles, angles 1 and 2 are alternate interior angles; so are angles 3 and 4.



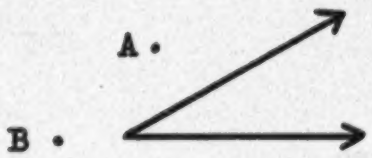
Corresponding angles, the pairs of corresponding angles in this figure are ∠ 1 and ∠ 2, ∠ 3 and ∠ 4, ∠ 5 and ∠ 8, ∠ 7 and ∠ 6.



Exterior angle, an angle like ∠ ACD pictured here.

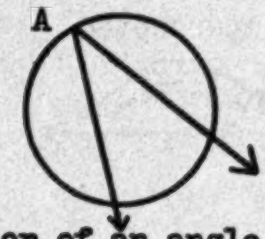


Exterior of an angle, points outside the angle; such as A, B, and C.



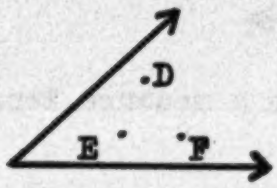
C

Inscribed angle, an angle formed by two rays meeting on the circle, with part of each ray being in the interior of the circle.

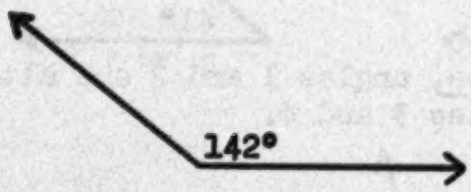


The angle A is inscribed in the circle.

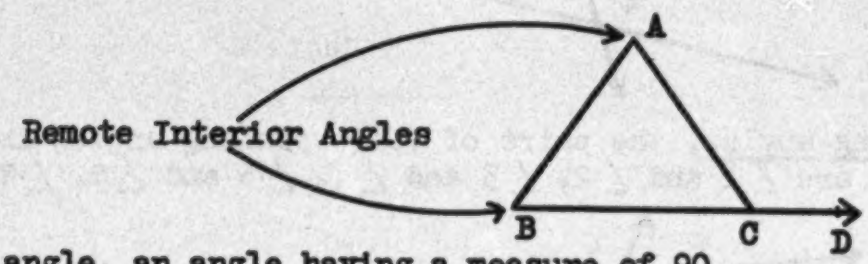
Interior of an angle, points inside the angle like D, E, and F.



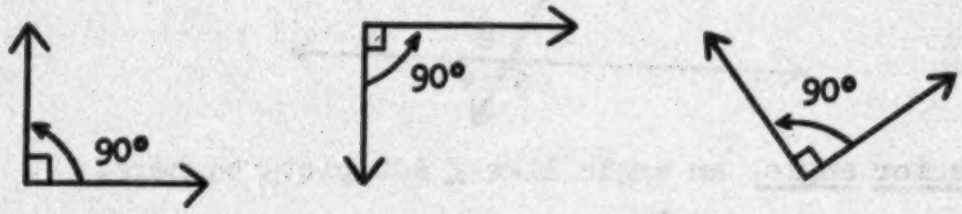
Obtuse angle, an angle with a measure greater than 90 but less than 180.



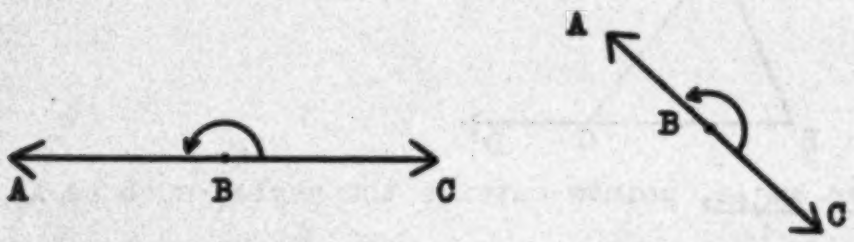
Remote interior angles, angle A and angle B are the two remote interior angles with respect to angle ACD of △ABC pictured here.



Right angle, an angle having a measure of 90.

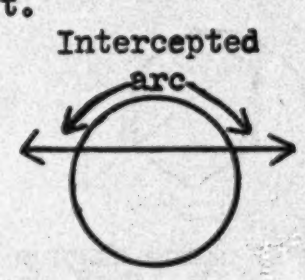


Straight angle, an angle having a measure of 180.

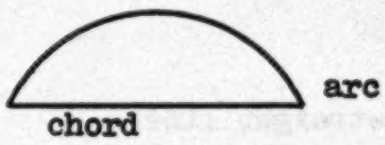


Arc, an unbroken part of a circle or curved line.

Intercepted Arc, a part of a circle or curved line which is separated from the whole by a line which crosses it.



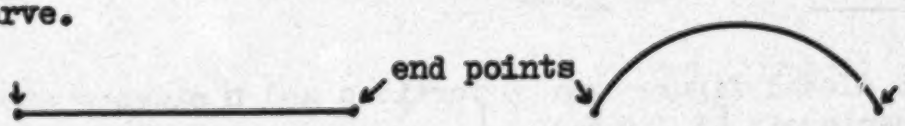
Chord, a line segment which cuts off an arc; the line segment from one end point of an arc to the other end point.



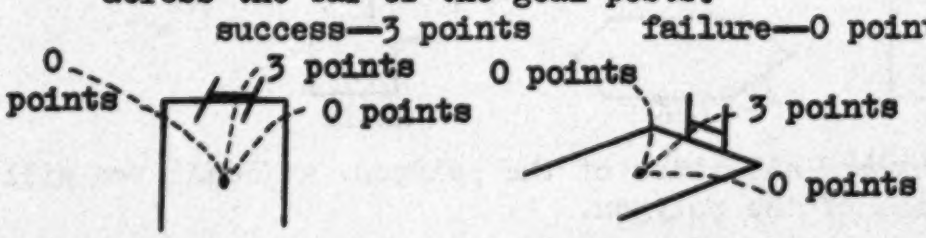
Collinear points, points that are on the same line.

Degree, unit used in measuring angles.

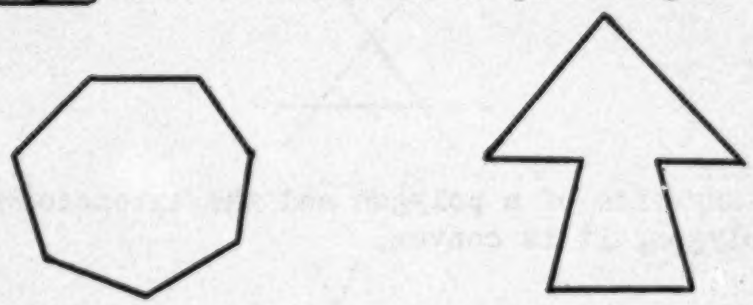
End points, the points on each end of a line segment or on a segment of a curve.



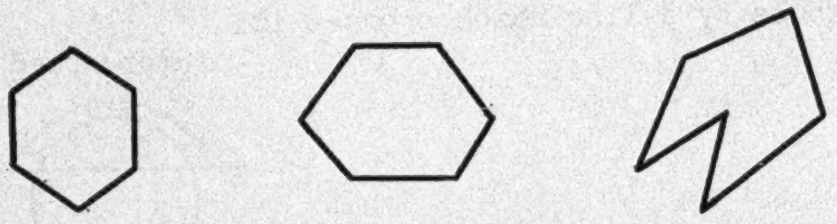
Field goal, a score made by kicking a football placed on the ground across the bar of the goal posts.



Heptagon, a seven sided closed plane figure.



Hexagon, a six sided closed plane figure.



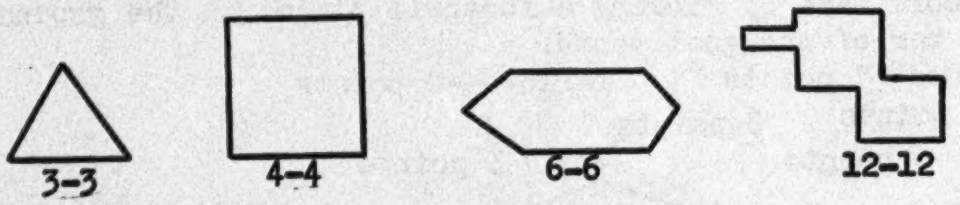
n-gon, a closed plane figure having n number of sides where n is any one of the elements of {3, 4, 5, 6, ...}

Noncollinear, not lying on the same straight line.
Example: Three or more points of a circle are noncollinear.

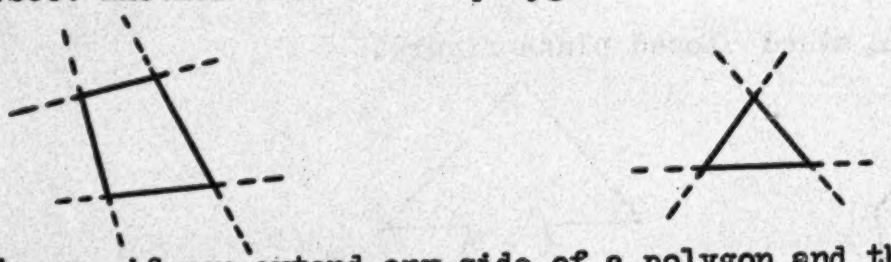
Octagon, an eight sided closed plane figure.

Pentagon, a five sided closed plane figure.

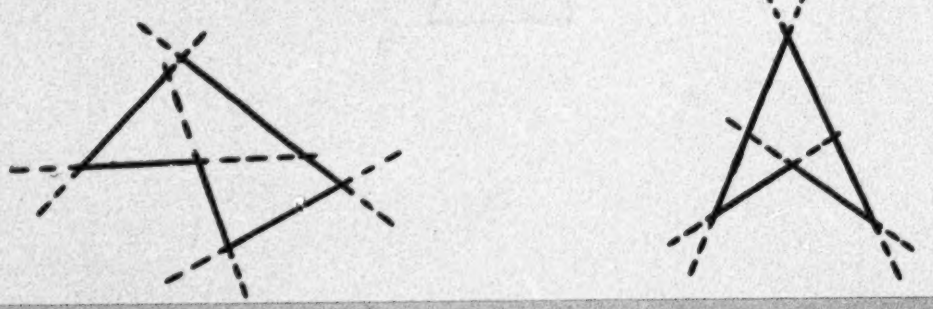
Polygon, a plane closed figure have n vertices and n sides where n is any one of the elements of {3,4,5,6, ...}



Concave Polygon, if you extend all sides of the polygon, at least two will intersect another side of the polygon.



Convex Polygon, if you extend any side of a polygon and the extension does does not go inside the polygon, it is convex.

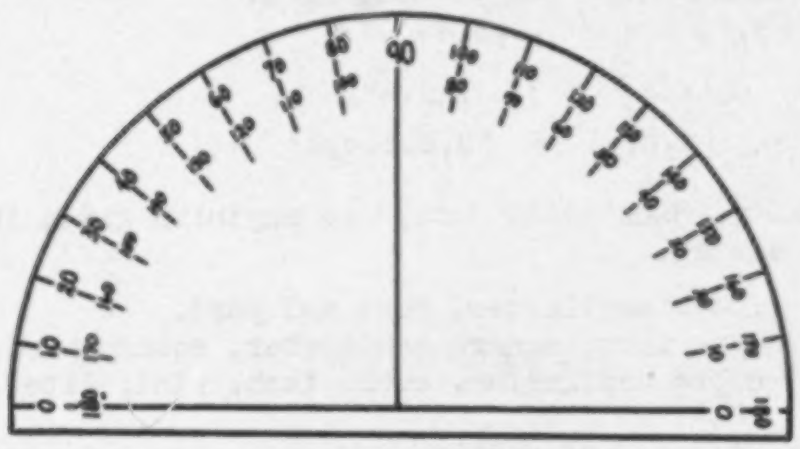


Regular Polygon, a polygon is regular if all the sides have the same length and all the angles have the same measure.

Parallel lines, lines that are an equal distance apart and never meet.

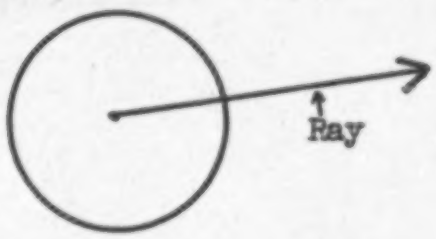
Polygonal region, the union of the polygon and its interior.

Protractor, an instrument for measuring angles in degrees.



Quadrilateral, a four sided closed plane figure.

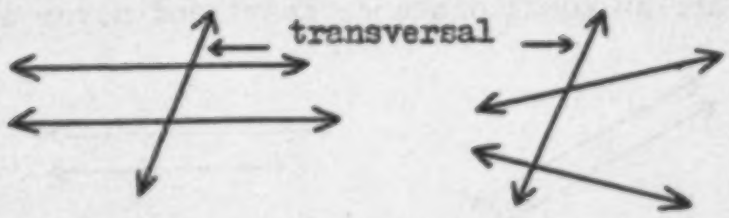
Ray, a straight line which begins at a point and continues unlimited in one direction.



Segment, a portion or a part.

line segment

Transversal, a line that intersects other lines.



Triangle, a three sided closed plane figure.

Triangular region, the union of the triangle and its interior.

Union, the union of sets A and B is a third set, C; such that the elements in set C are contained in at least one of sets A, B.

$\{1, 2\} \cup \{3,4\} = \{1,2,3,4\}$

$\{\square\} \cup \{\bigcirc,\triangle\} = \{\square,\bigcirc,\triangle\}$

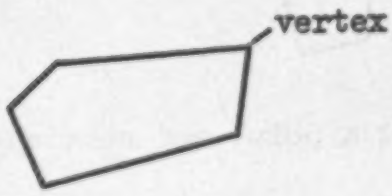
$\{a,e,i,o\} \cup \{i,o,u\} = \{a,e,i,o,u\}$

Unit, comes from the word unity which means "one," so anything given the measure of 1 is called a unit.

Some linear units are: inch, centimeter, foot and yard.
Some area units are: square inch, square centimeter, square yard.
Some volume units are: cubic centimeter, cubic inch, pint, liter and quart.
Some angular units are: degree and radian.

Vertex, is (1) the place where two sides of flat figure meet;
(2) the place where three or more sides of a solid figure meet.

(1)



(2)

These figures have five vertices.

3252
5f.

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